Resonance and Capture of Jupiter Comets

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Acknowledgements

- **H. Poincaré, J. Moser**
- **C. Conley, R. McGehee**
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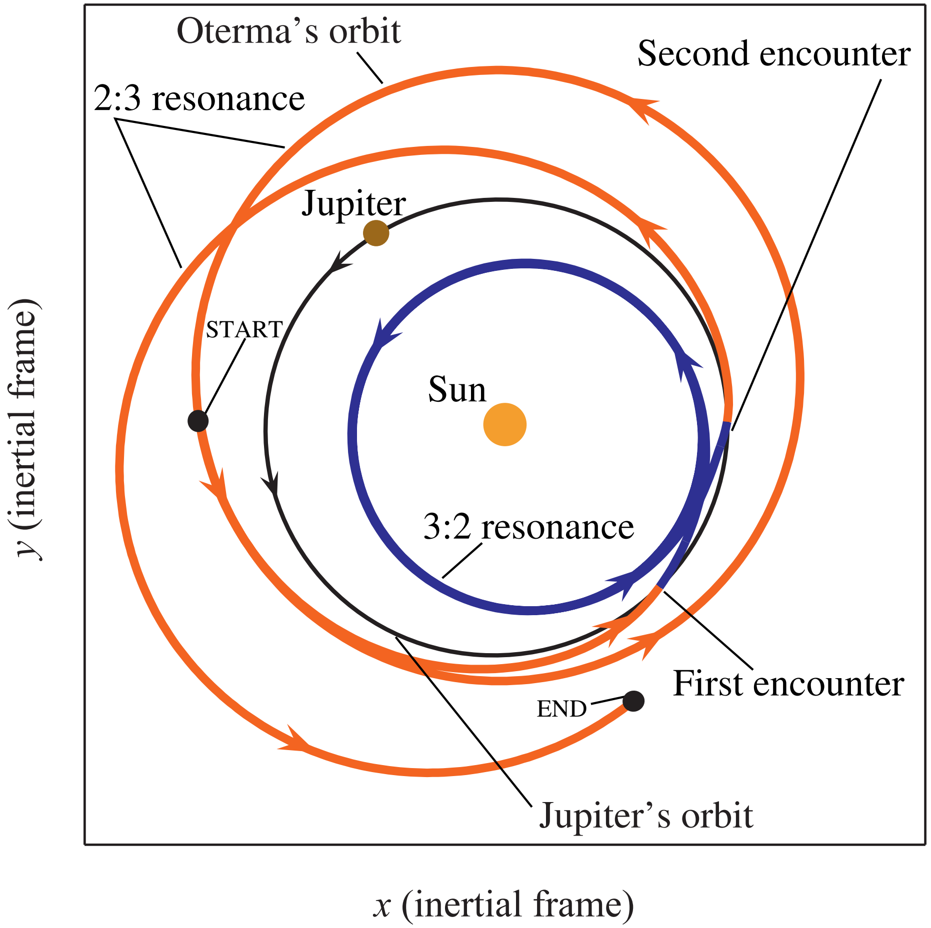
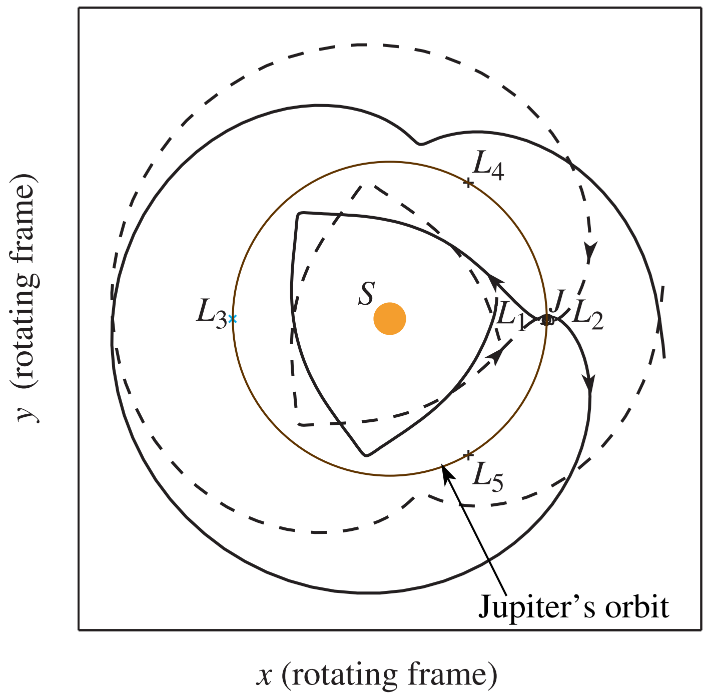
■ Jupiter Comets: e.g., *Oterma*

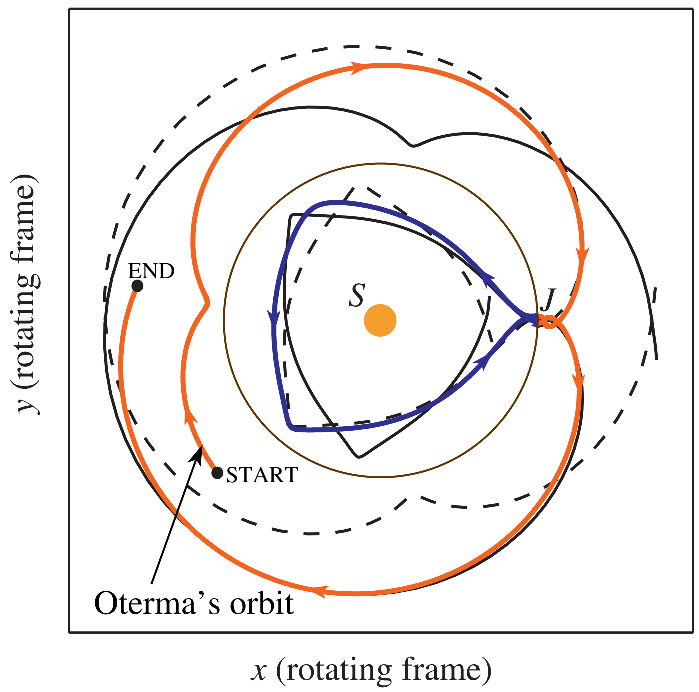
- ***Rapid transition*** from **outside** to **inside** Jupiter's orbit.
- ***Captured temporarily*** by Jupiter during transition.
- **Exterior** (**2:3 resonance**). **Interior** (**3:2 resonance**).



Previous Work

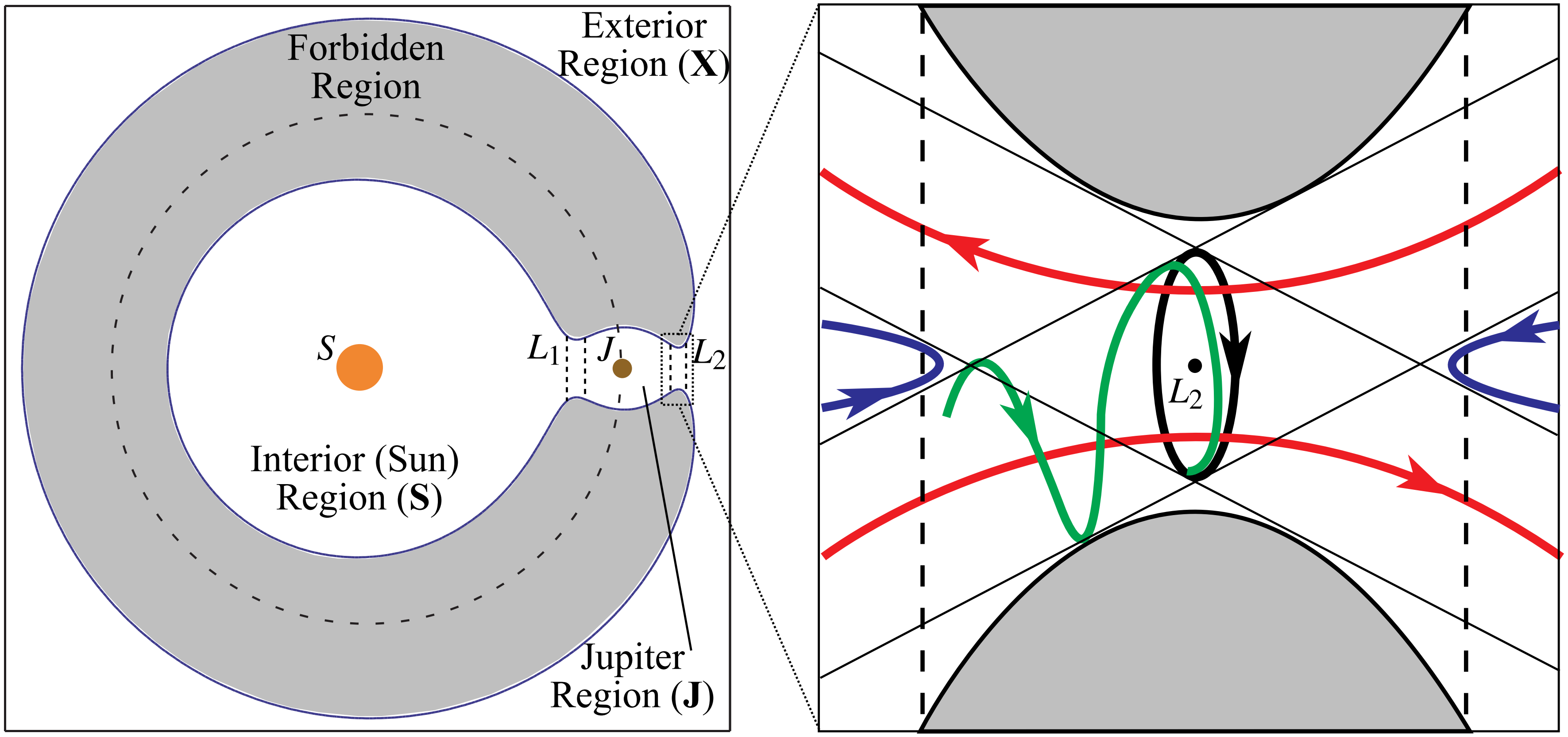
- Belbruno/B. Marsden [1997]
- Lo/Ross [1997] : Comets pass by L_1 & L_2.
  - In Sun-Jupiter ***rotating frame***, comets follow ***stable & unstable invariant manifolds***.
- Works by Moser/Conley/McGehee. Llibre/Martinez/Simó [1985].





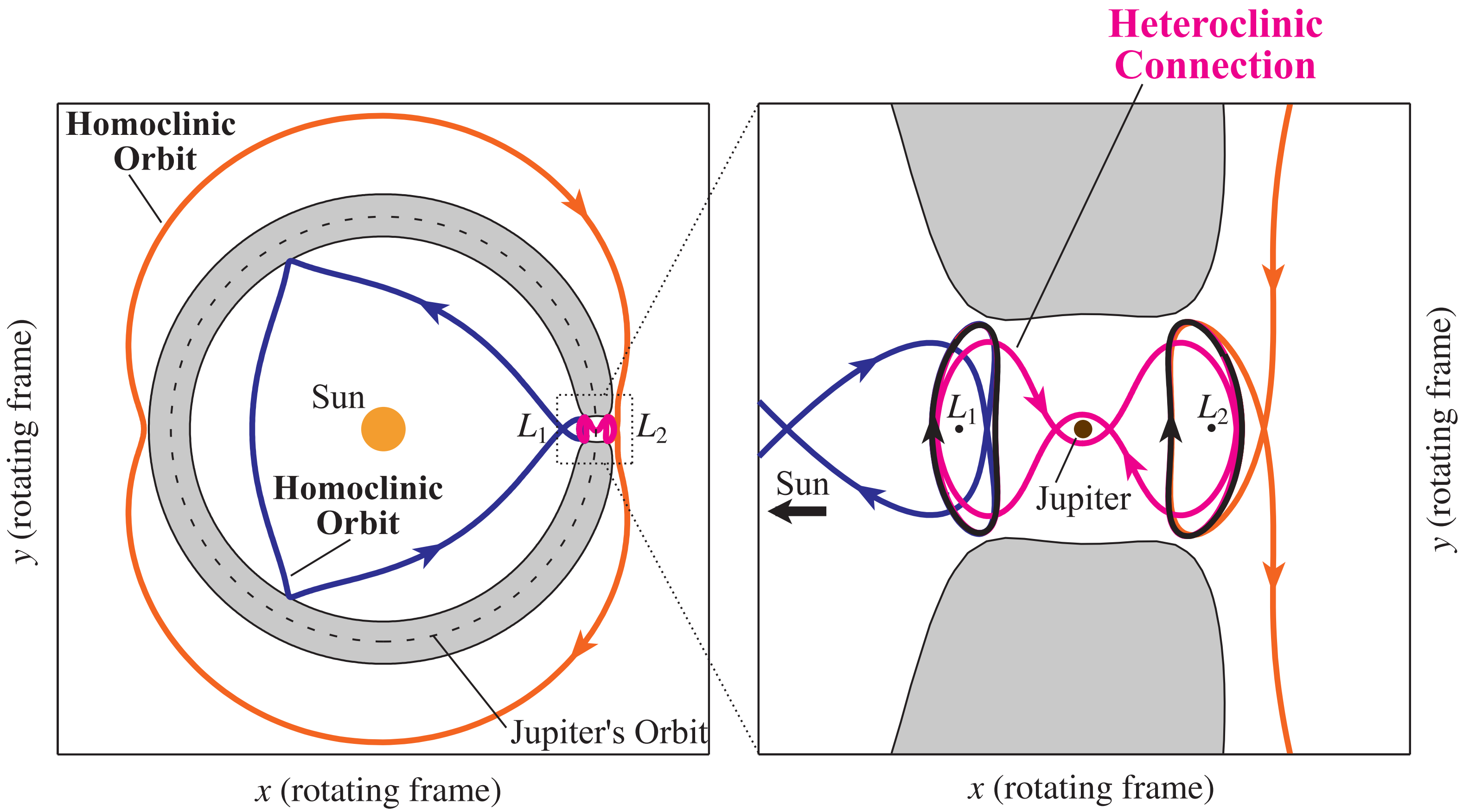
■ Flow Near L_1 and L_2

- ***Energy value*** $> L_2$: ***Hill's region*** has ***neck*** about L_1 & L_2.
- Comet makes transition through ***equilibrium region*** necks.
- Four orbit types: periodic, **asymptotic**, **transit** & **nontransit**.



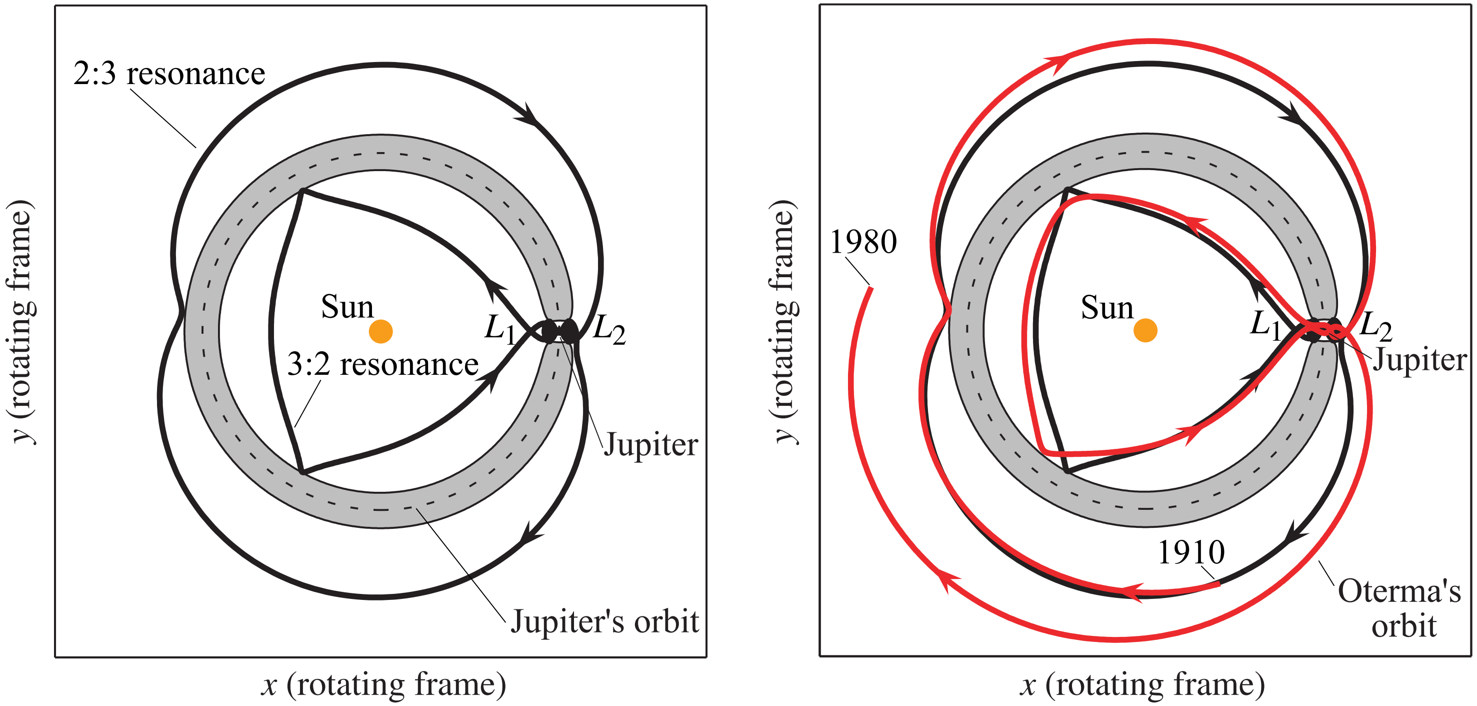
Jupiter Comets Use Heteroclinic Connection

- **Heteroclinic connection** between L_1 & L_2 periodic orbits.
  - Link with homoclinic orbits to make ***chain***.
- Comets follow dynamical ***channels*** for rapid transition.



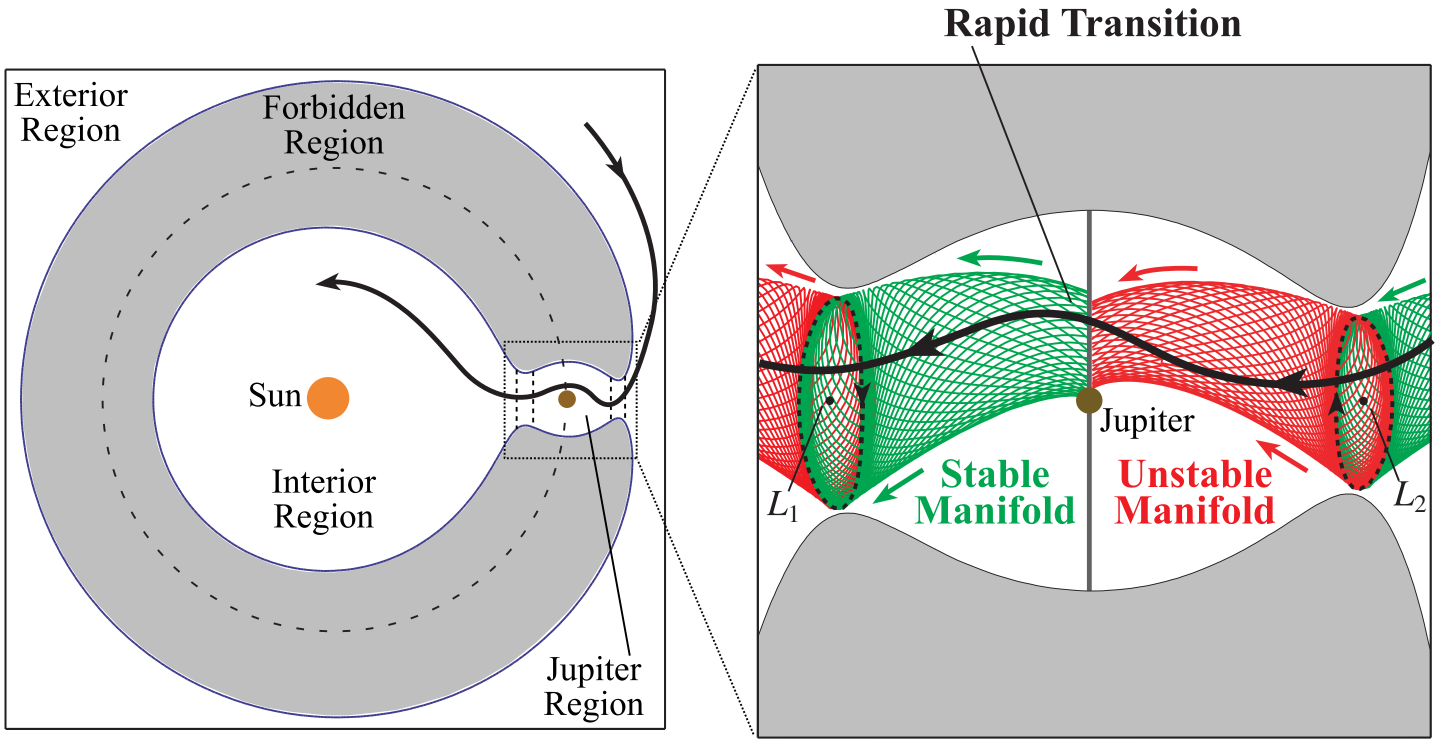
Jupiter Comets: Following Dynamical Channels

- Consider comet ***Oterma*** from 1910 to 1980.
  - Determine energy during transition.
  - Compute **homoclinic-heteroclinic chain**.
  - Overlay **chain** with ***Oterma's*** orbit (at right).

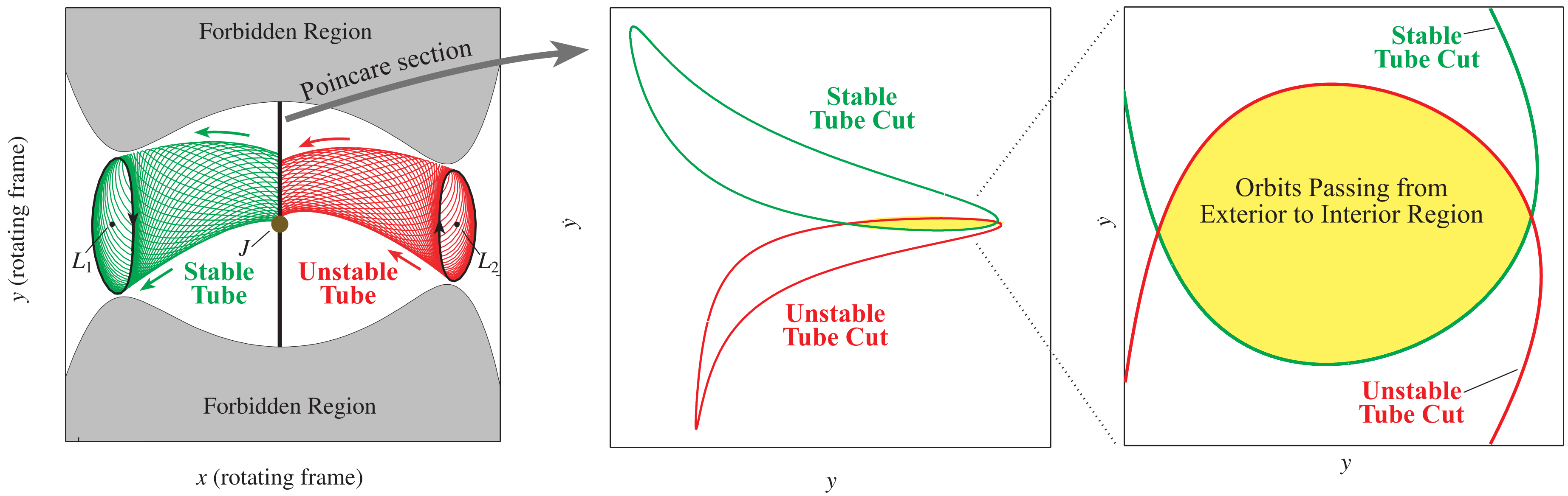


Jupiter Comets: Rapid Transition Mechanism

- **Rapid transition** between exterior/interior via
  - **stable** & **unstable** manifold ***tubes*** which contain **transit** orbits.

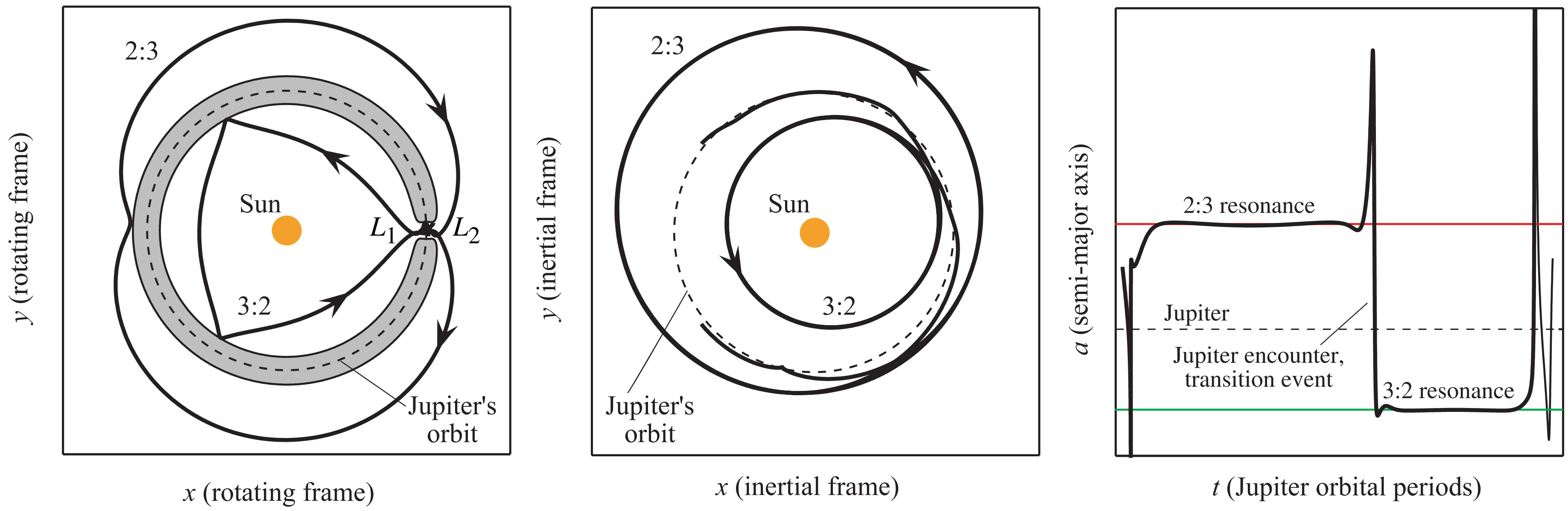


- Jupiter region Poincaré section:
  - L_2 **unstable tube** intersects L_1 **stable tube**.
  - Contains exterior $\longrightarrow$ interior orbits.

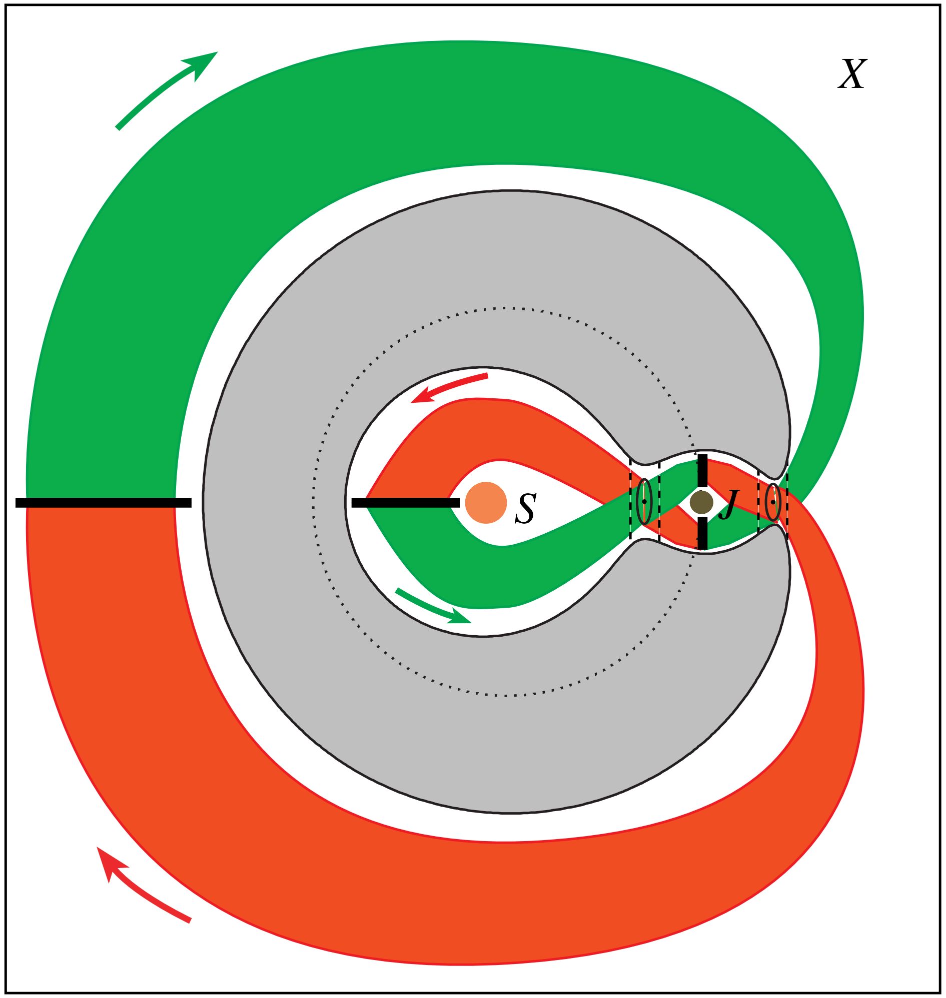


Tubes and Resonance Transition

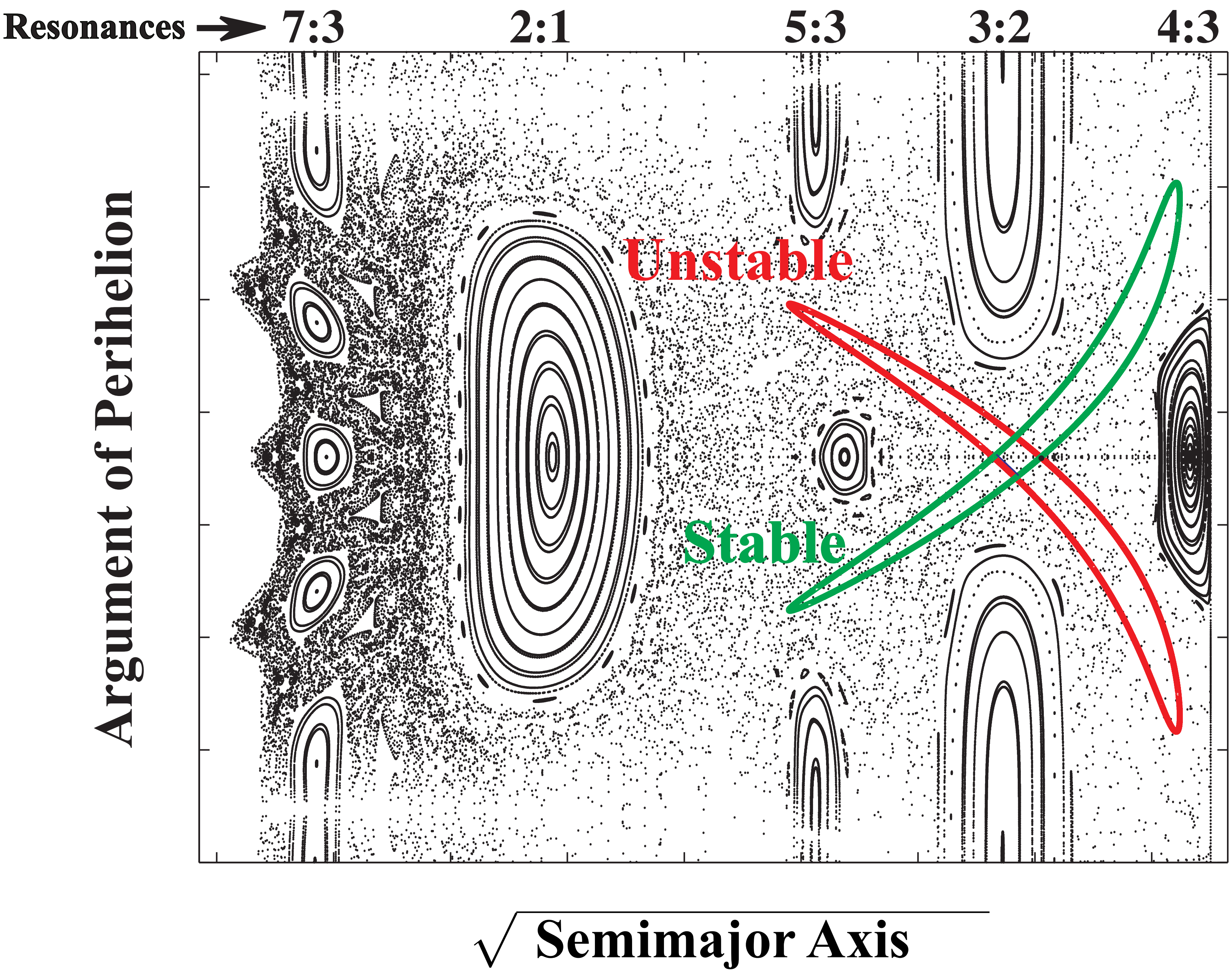
- Tubes are ***transport mechanism*** connecting interior and exterior Hill's regions.
- Connect **mean motion resonances**.
  - e.g., ***Oterma's* 2:3$\longrightarrow$3:2** transition.
- Can construct ***Oterma***-like transition.



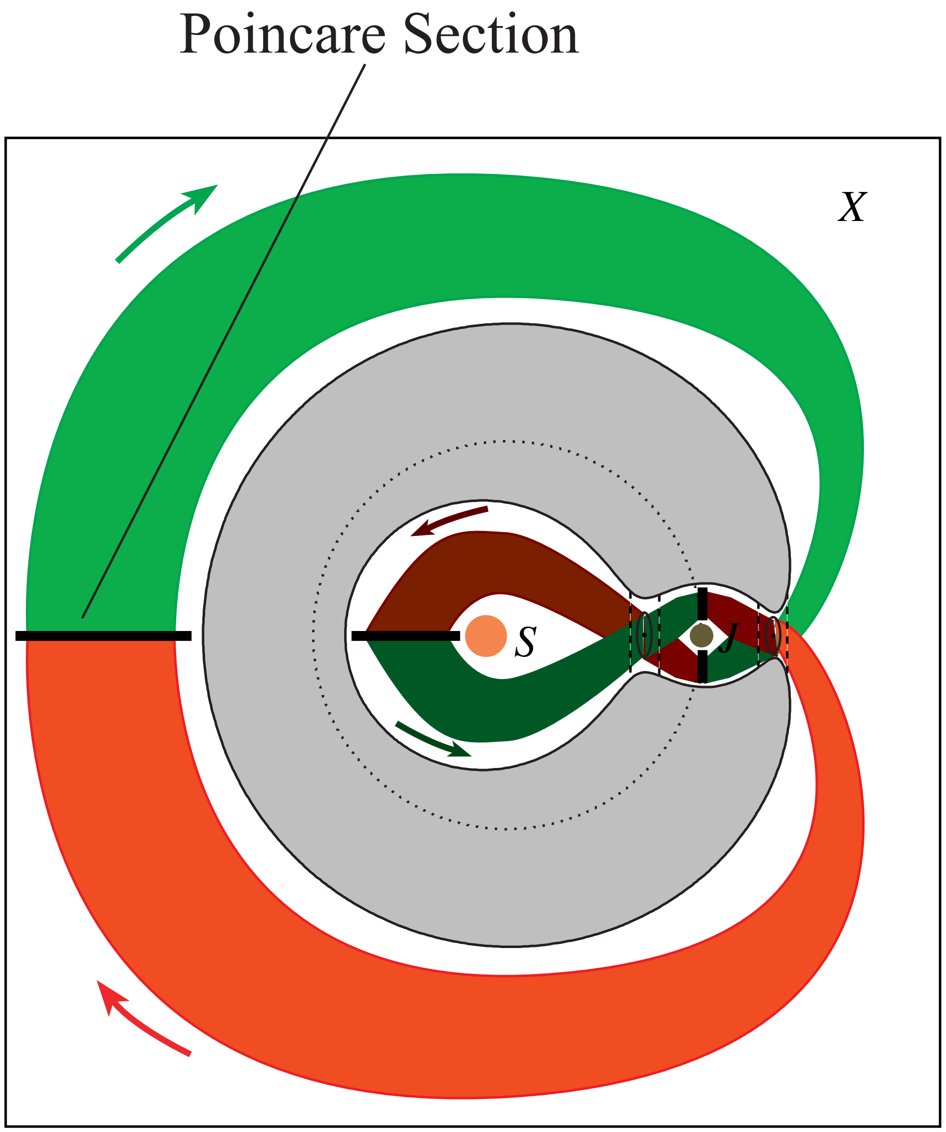
- Schematic of tube location. Poincaré sections are **solid lines**.
  - **Stable** manifolds in **green**,
  - **Unstable** manifolds in **red**.



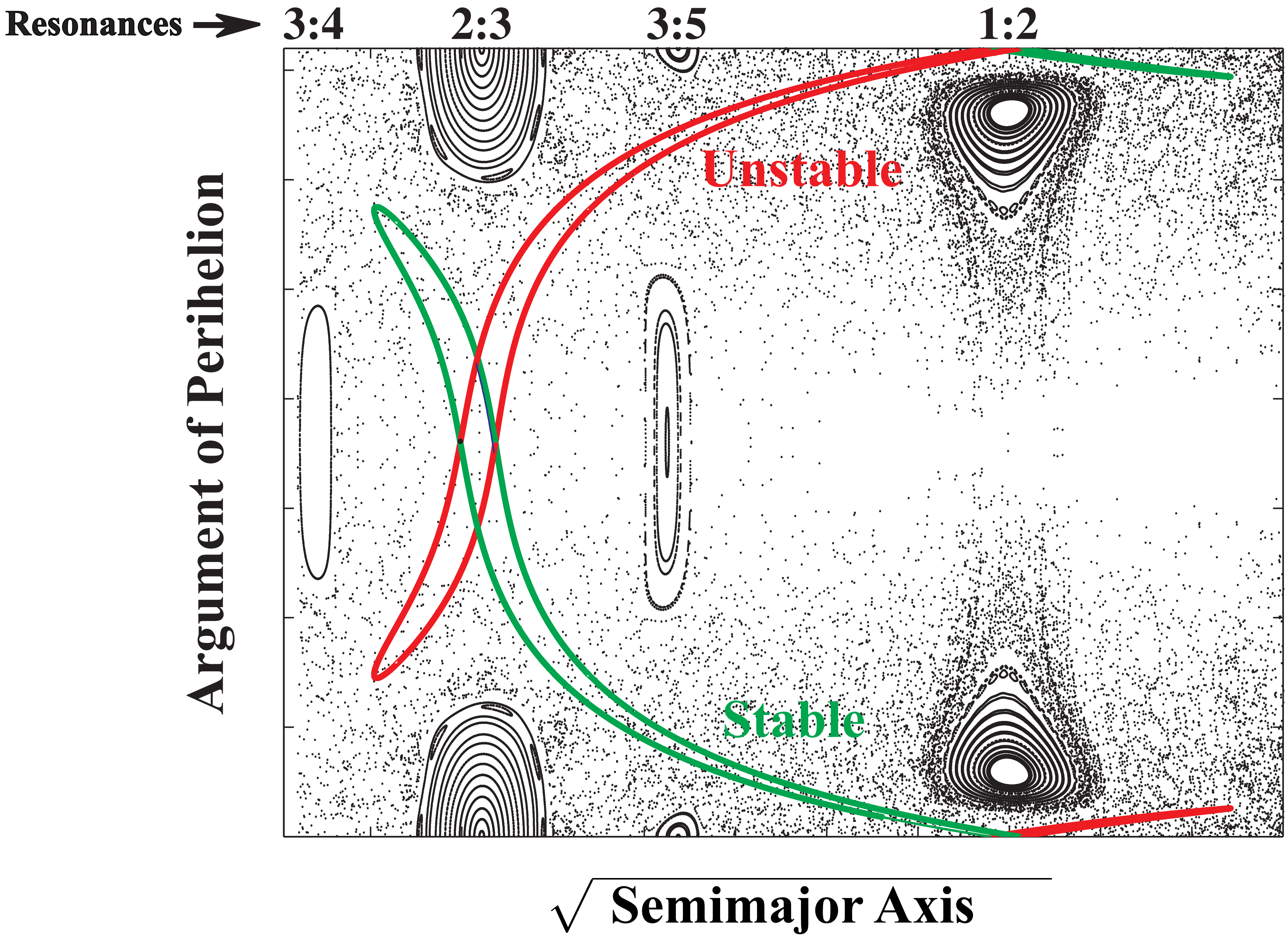
- Interior region Poincaré section.



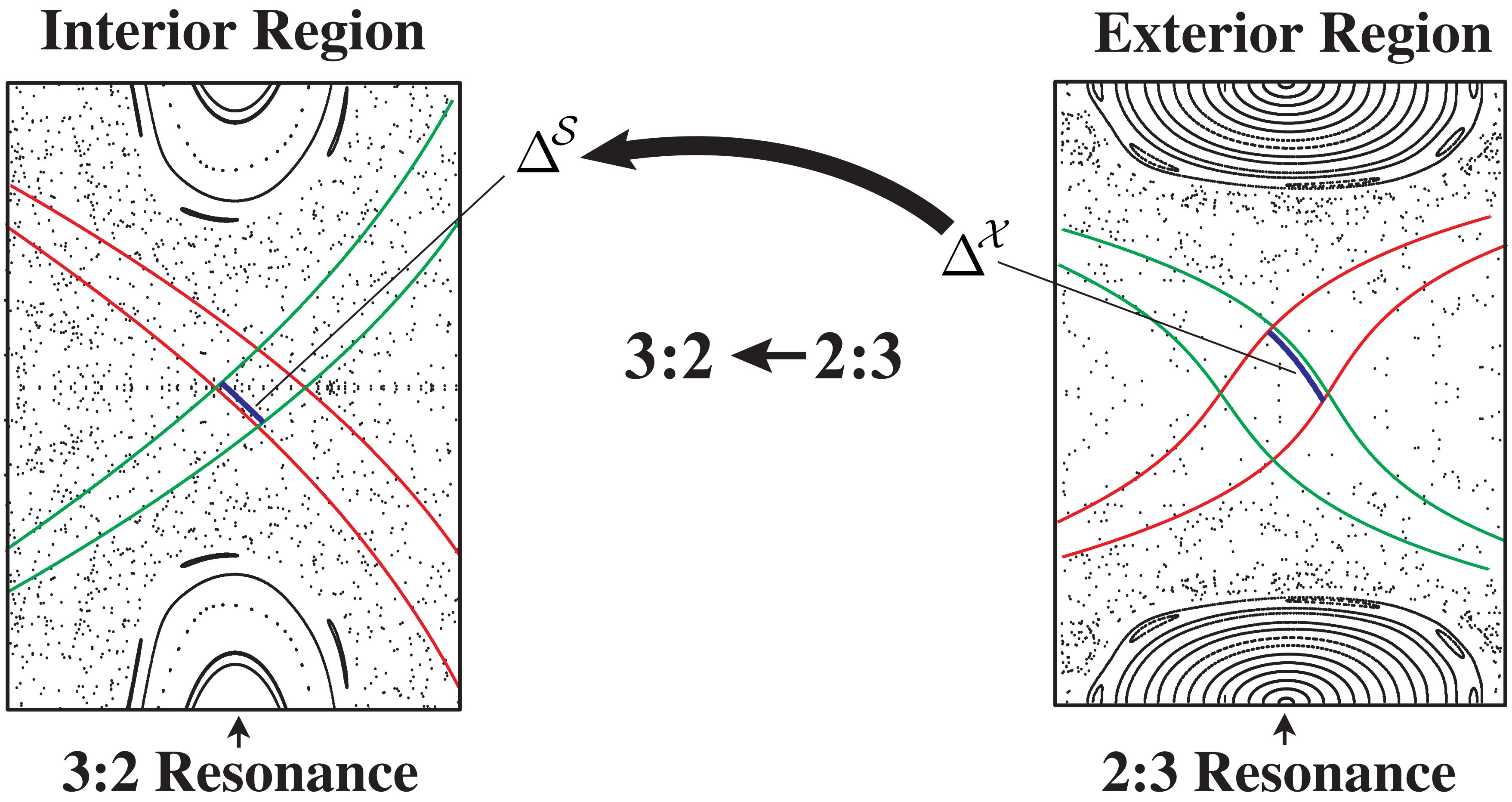
- Look at Poincaré section of tubes in ***exterior region***.
  - L_2 orbit **stable** & **unstable** manifolds.



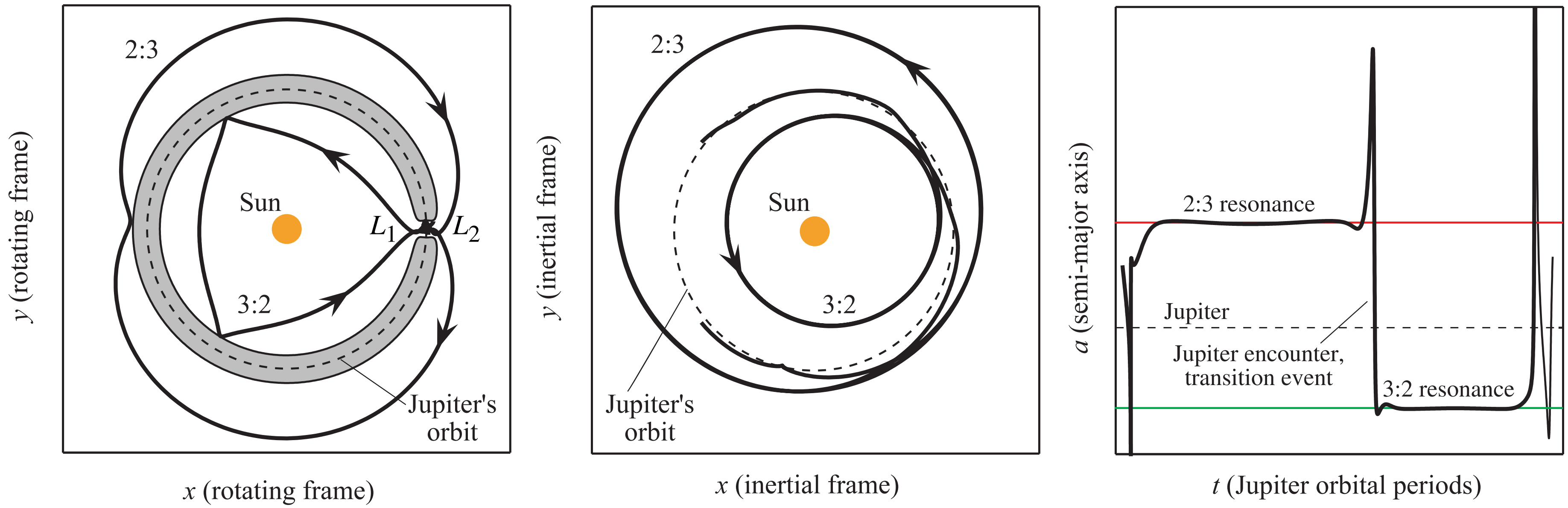
- Exterior region Poincaré section.



- Transition resonances for ***Oterma's*** energy:
  - Interior: **3:2**
  - Exterior: **2:3**
- **2:3**⟶**3:2** connected via Jupiter region.

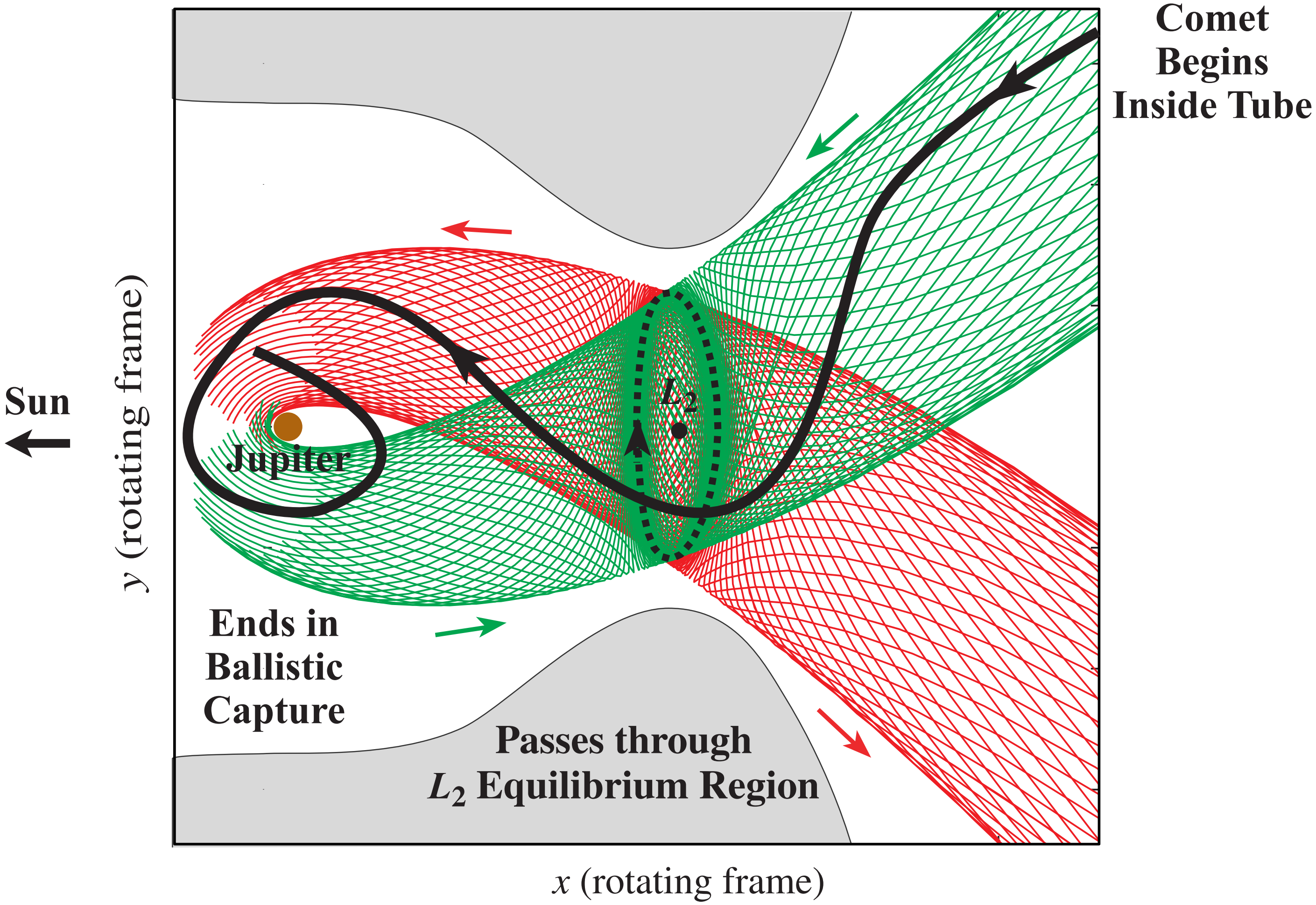


- Pick initial condition in **2:3⟶3:2** channel.
  - Forward/backward integrate ***Oterma***-like resonance transition.



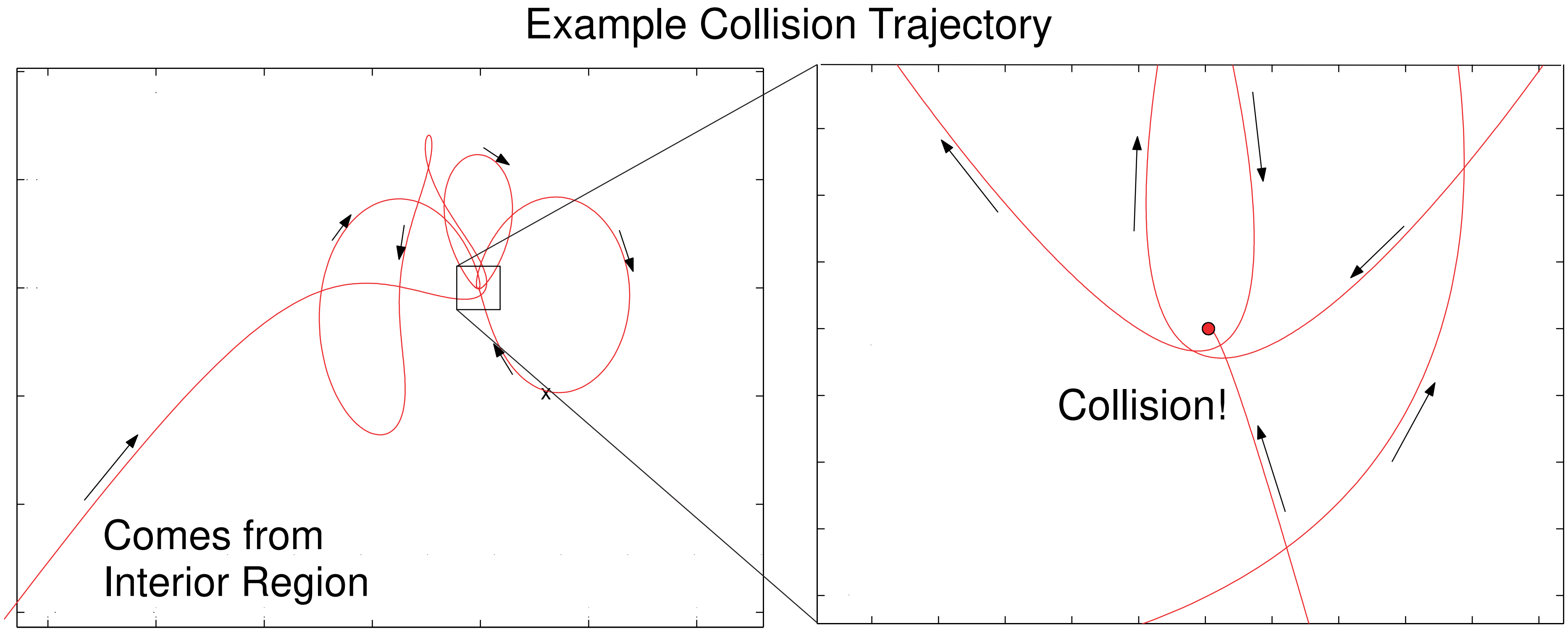
Temporary Capture Around Jupiter

- Must come into Jupiter region via tubes for capture.
- Several revolutions possible before departing.

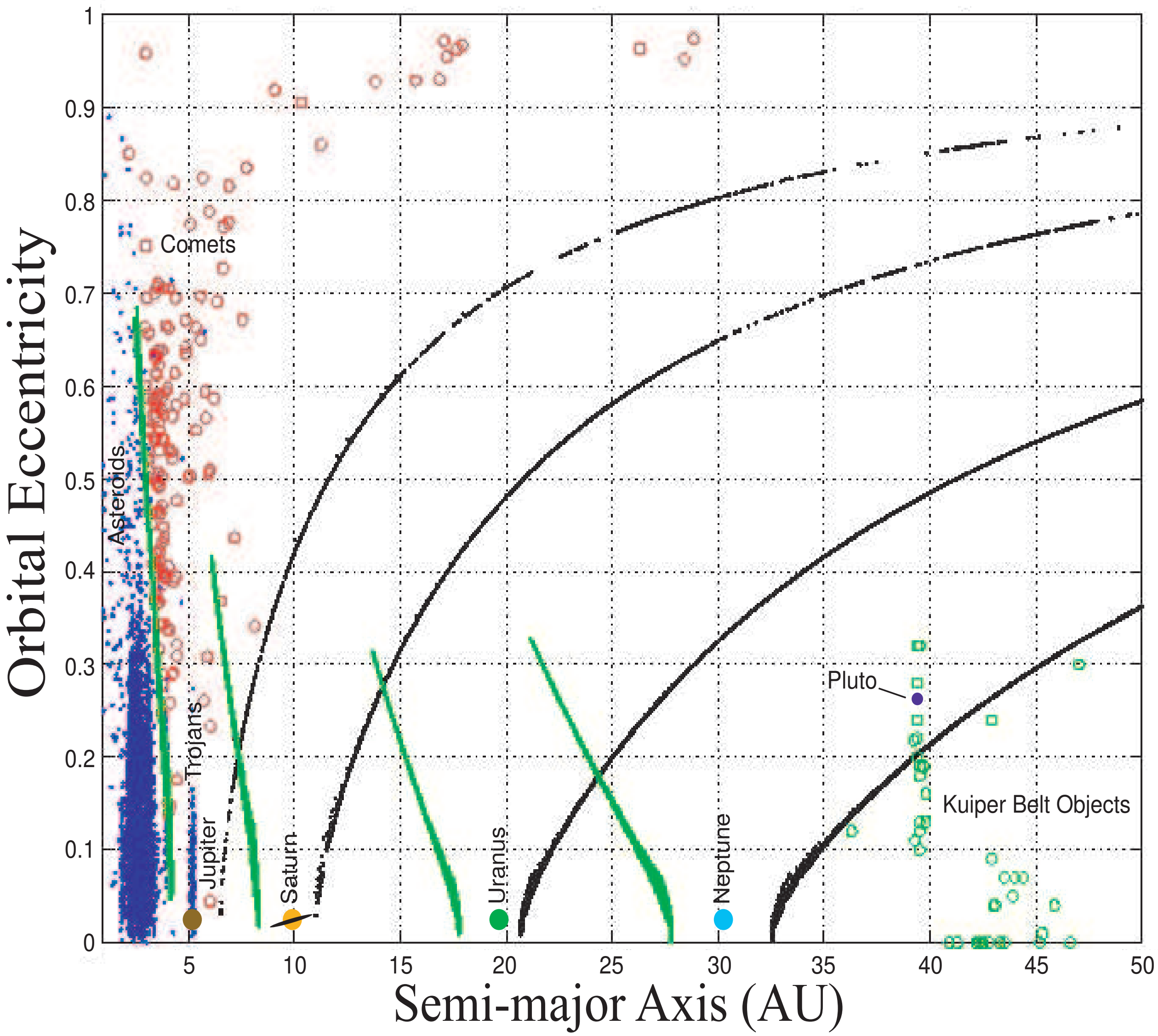


Collision with Jupiter

- Some portion of tube intersects Jupiter.
  - e.g., ***Shoemaker-Levy 9*** simulation

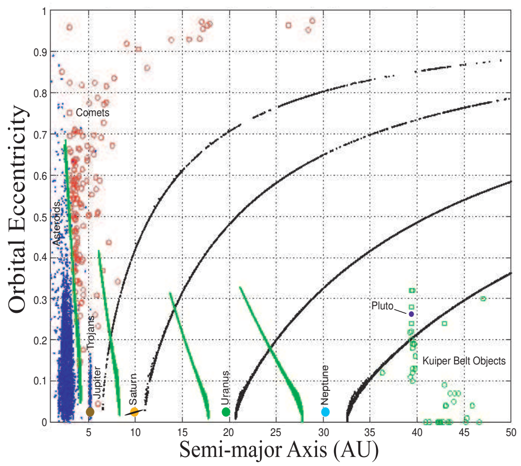


Further Work: Transport Throughout Solar System

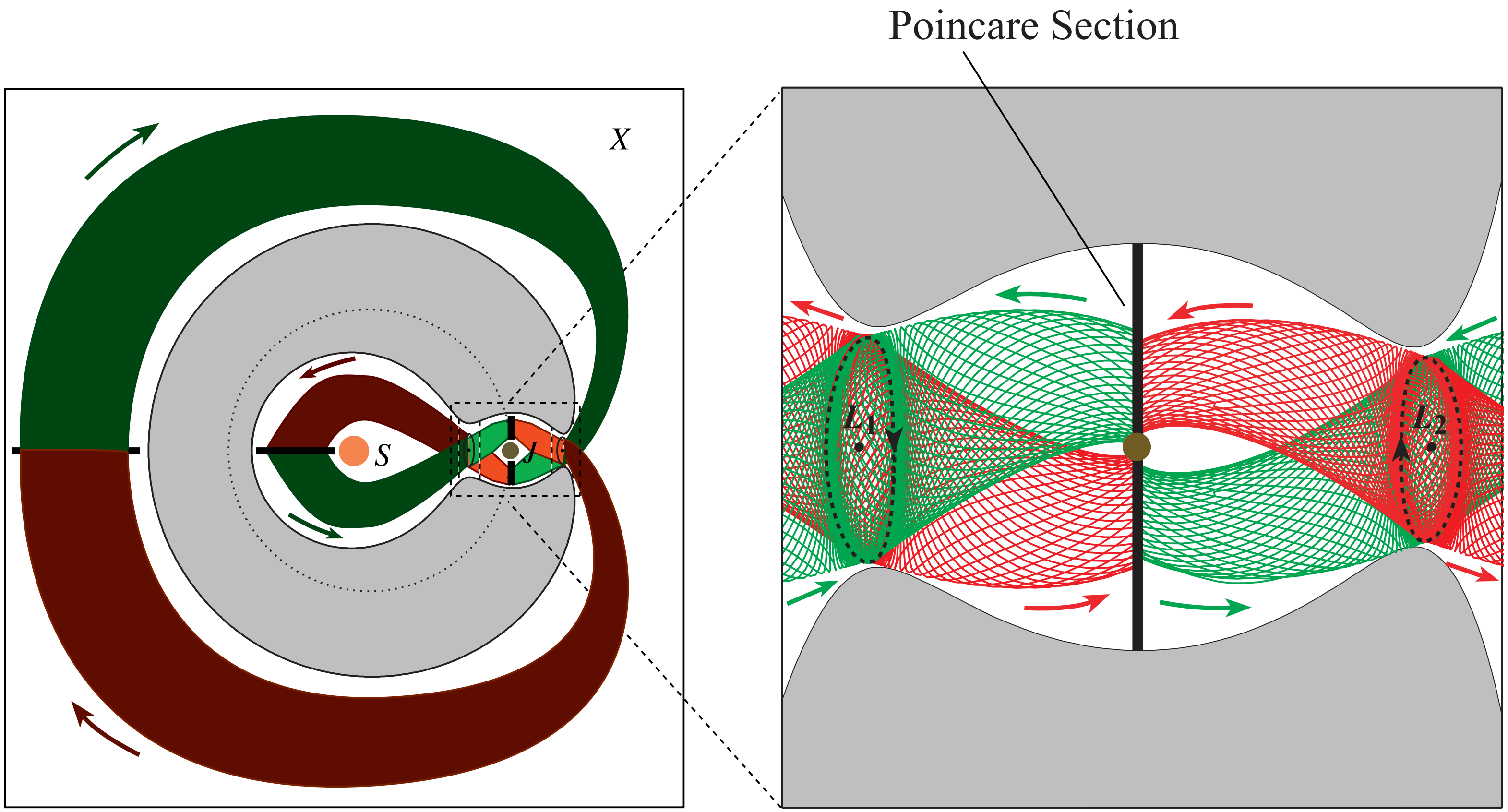


More Information and References

- **Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross** ***Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics***, **Chaos, vol. 10(2) [2000], pp. 427–469**;
  - **http://www.cds.caltech.edu/~marsden/**
  - Click on "current issue" of **http://ojps.aip.org/chaos/**
- Email: **shane@cds.caltech.edu**



- Look at Poincaré section of tubes in ***Jupiter region***.
    - L_2 orbit **unstable** $\longrightarrow$ L_1 orbit **stable** manifolds.



- Jupiter region Poincaré section:
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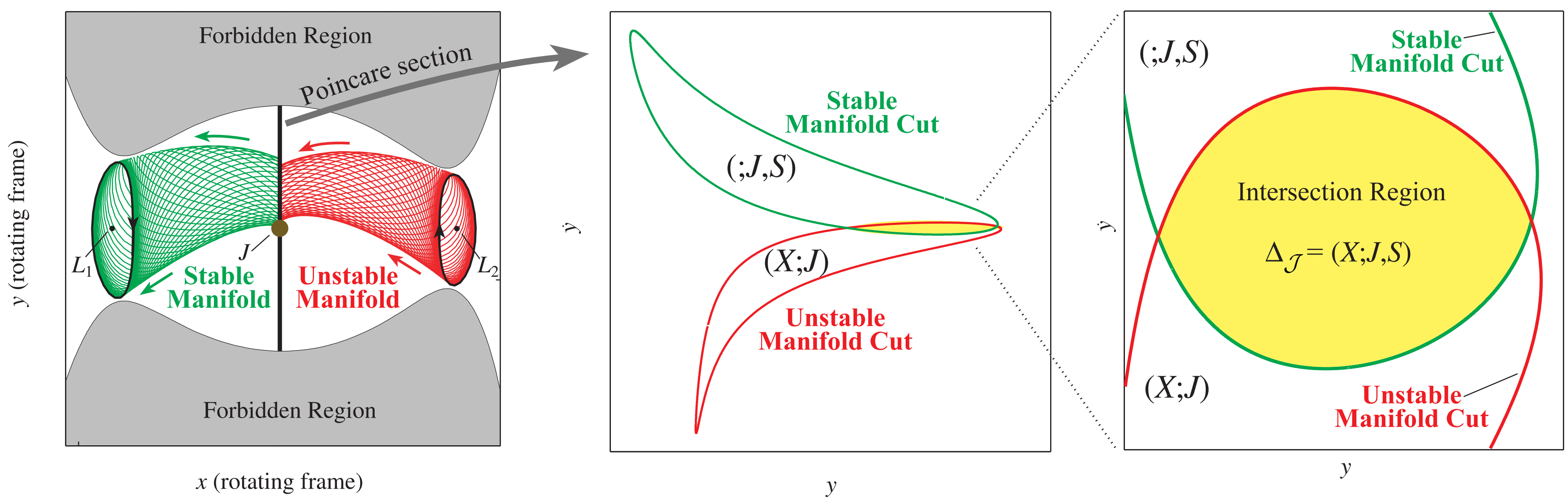
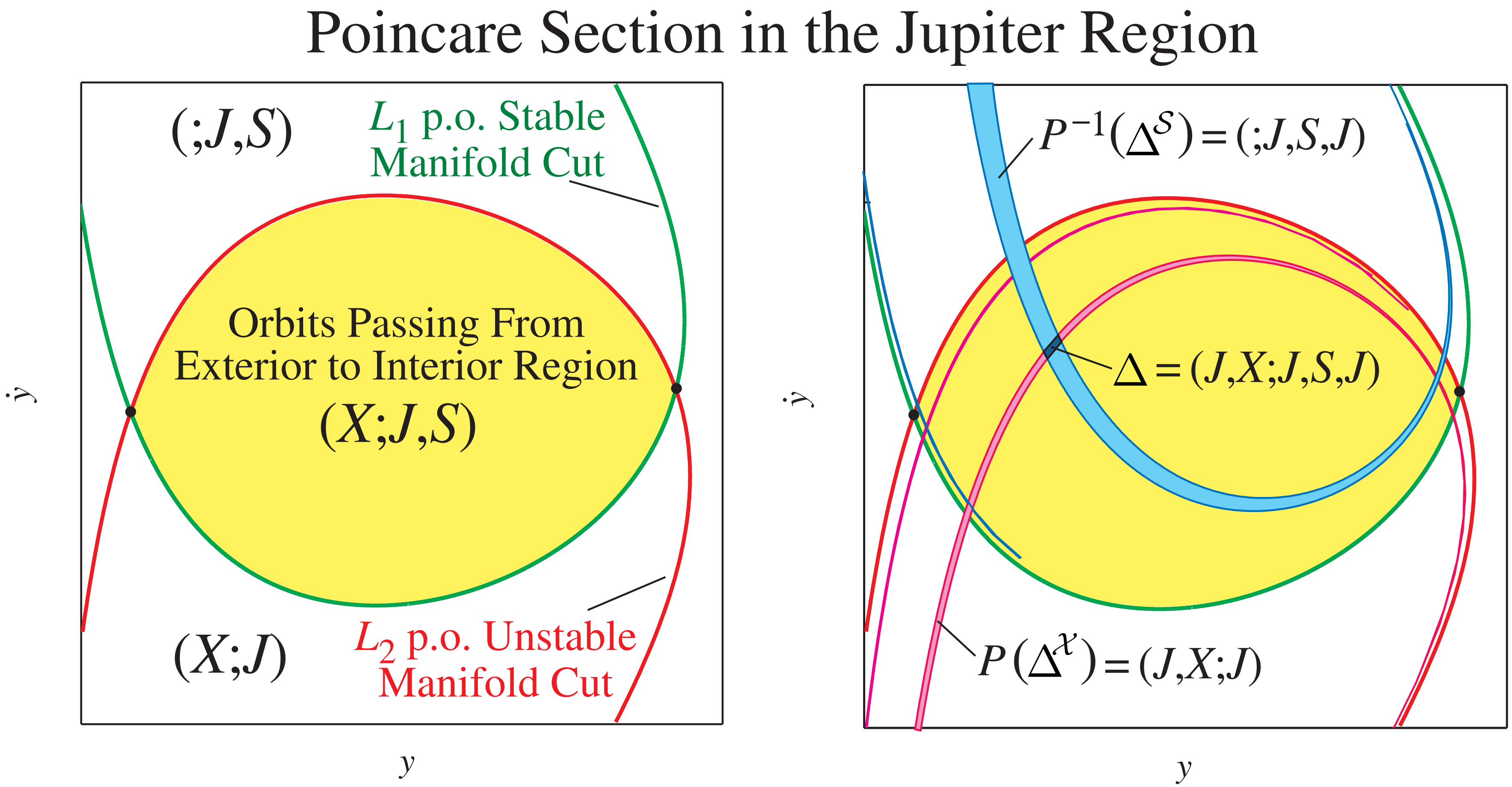


- Intersection region contains transit orbits:
  - exterior $\longrightarrow$ interior



Connection Between Interior/Exterior Resonances

- Δ contains orbits in **2:3** $\longrightarrow$ **3:2** ***transition***.
- Comets, e.g. ***Oterma***, pass through such regions.